CURRICULUM
EDUCATION LIBRARY

CURRICULUM GUIDE
GRADE IX MATHEMATICS

(INTERIM)

CURRICULUM
QA
14
C22
A3
A3153
1967

ALTA
510
1967
Gr9

CURRGDHT

CURR

MENT OF THE PROVINCE OF ALBERTA
MENT OF EDUCATION

SEPTEMBER, 1967

PREFACE

This statement has been prepared under the direction of the Junior High School Mathematics Subcommittee of the Department of Education, Alberta, as a guide for teachers using either of the authorized texts.

Teachers who have had experience with these texts during the experimental period have generously provided time and effort to the preparation of this bulletin. The intention is not to repeat materials included in either the Teacher's Edition or the Teaching Guide, but rather to offer practical suggestions based on the experiences of these teachers.

ACKNOWLEDGEMENT

The Department of Education acknowledges with appreciation the contributions of the following committee members, subcommittee members and teachers of mathematics, to the preparation of this Curriculum Guide for Grade Nine Mathematics.

Junior High School Curriculum Committee

M. L. Watts	Director of Curriculum, Chairman
Dr. J. S. Hrabi	Associate Director of Curriculum
O. Fadum	Superintendent of Schools, Olds
Dr. T. G. Finn	Associate Dean, Faculty of Education, U. of C., Calgary
J. D. Harder	Supervisor of Industrial Arts
H. I. Hastings	Elementary School Consultant
W. Hughes	Alberta Teachers' Association, Edmonton
Miss A. B. MacFarlane	Supervisor of Home Economics
Carl Poloway	Alberta Teachers' Association, Westlock
J. H. Finlay	Director, Junior High Schools, Edmonton Public School Board
F. L. Wilcox	Assistant Superintendent of Secondary Schools, Calgary
Helen Pierce	Alberta Teachers' Association, Camrose
Mrs. P. Gallagher	AFHSA, Brant
R. A. Morton	Associate Director of Curriculum
I. Goresky	Associate Director of Curriculum, secretary

Junior High School Mathematics Subcommittee

R. P. Plaxton	Ass't Principal, Vincent Massey Junior High School, Calgary, Chairman
Dr. A. A. Gibb	Professor, University of Calgary, Calgary
Dr. J. S. Hrabi	Associate Director of Curriculum
G. P. Jepson	Principal, Colonel Walker School, Calgary
L. Kunelius	Inspector of High Schools, Calgary
Dr. S. A. Lindstedt	Professor, Faculty of Education, U. of C., Calgary
S. Mallet	Principal, Central Junior High School, Red Deer
Dr. D. Nelson	Associate Professor, Faculty of Education, U. of A., Edmonton
J. Wood	Assistant Supervisor of Junior High Schools, Edmonton
L. Pallesen	Assistant Superintendent, Secondary Schools, Calgary, Secretary
N. Chamchuk	Superintendent of Schools, St. Paul

Special acknowledgement is made to the following teachers who have contributed much in the preparation of the Curriculum Guide.

C. Armstrong	Parkview Junior High School, Edmonton
R. Anderson	Vincent Massey Junior High School, Calgary
Mrs. Ella Freeman	Ponoka Junior High School, Ponoka
Mrs. M. Herchek	Allendale Junior High School, Edmonton
J. Fabian	Wellington Junior High School, Edmonton

TABLE OF CONTENTS

Page

Preface i

Acknowledgement ii

I. OBJECTIVES 1

II. AUTHORIZATIONS FOR GRADE IX MATHEMATICS 2

III. SUPPLEMENTARY REFERENCES 2

IV. INTRODUCTION 3

1. Definitions 3
2. Symbolism 3
3. Sequential Nature of Development 3
4. Emphasis on Understanding 4
5. Need for Continuous Drill 4
6. Lesson Preparation 4
7. Visual and Manipulative Material 5

V. EXPLORING MODERN MATHEMATICS, BOOK III 7

A. General Considerations 7

1. Teacher's Edition 7
2. Let's Explore Exercises 7
3. Programmed Supplement 8
4. Sequence of the Text 8
5. Proofs 8
6. Applied Problems 8
7. Review Exercises 8
8. Tests 8
9. Definitions 9
10. Further Comments 9

B. Minimum Program, Time Schedule, Optional Content 10

1. Minimum Program 10
2. Time Schedule 10
3. Optional Content 10

C. Lesson Comments 11
Chapter 1 11
Chapter 2 14
Chapter 3 16
Chapter 4 18
Chapter 5 21
Chapter 6 21
Chapter 7 25
Chapter 9 27

VI. SEEING THROUGH MATHEMATICS, BOOK III SPECIAL EDITION 28
General Considerations
1. Semi-Programmed Nature 28
2. Minimum Program and Schedule of Units 28
Lesson Comments 33
Unit 13 33
Unit 15 35
Unit 16 38
Unit 17 38
Unit 18 40
Unit 19 42
Unit 20 45
Units 21 and 22 47

OBJECTIVES

The objectives for mathematics are:

1. To develop an understanding of mathematical concepts.
2. To develop skill in the use of the fundamental processes.
3. To develop systematic methods of analyzing problems and of presenting their solutions.
4. To develop habits of precise thought and expression.
5. To develop an understanding of the significance and application of mathematics in the modern world.

AUTHORIZATION FOR GRADE IX MATHEMATICS

Effective September, 1967, **either** of the textbooks listed below may be selected for use in Grade IX classrooms.

1. *Exploring Modern Mathematics, Book 3*, Keedy, Jameson, Johnson, Purvis and Atkinson (Holt, Rinehart and Winston of Canada, Ltd.).

2. *Seeing Through Mathematics, Book 3, Special Edition*, Van Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage & Co.).

SUPPLEMENTARY REFERENCES

1. *Teacher's Edition, Exploring Modern Mathematics, Book 3,* Keedy, Jameson, Johnson, Purvis and Atkinson (Holt, Rinehart and Winston of Canada, Ltd.).

2. *Tests for Exploring Modern Mathematics, Book 3* (Holt, Rinehart and Winston of Canada, Ltd.).

3. *Charting the Course for Arithmetic,* Hartung, Van Engen, Knowles and Gibb (Scott, Foresman and Company).

4. *Response Verification Book, Seeing Through Mathematics, Book 3, Special Edition, V*an Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage and Co.).

5. *Teaching Guide, Seeing Through Mathematics, Book 3, Special Edition,* Van Engen, Hartung, Trimble, Berger, Cleveland and Evenson (W. J. Gage and Co.).

INTRODUCTION

To teach Grade IX Mathematics using either of the texts authorized for the first time in Alberta in September, 1967 it is essential that the teacher be familiar not only with the general content, but also with the detail of the texts used in Grades VII and VIII. In the ideal situation the Grade IX teacher will have taught both the Grade VII and Grade VIII courses. It would also be desirable for the teacher to spend some time becoming familiar with the content of the Grade V and VI programs, so that the total background of the students is known. Only where a teacher devotes time and effort to secure such background knowledge, can instruction be given with maximum effectiveness and teacher confidence.

More than ever before, a detailed knowledge of the subject matter of preceding grades is essential if instruction is to be effective. Some of the important features of the programs being implemented through the new authorizations are listed below.

1. Definitions

Definitions which differ from traditional usage, and which may differ from one authorization to the other, have been introduced and are rigorously followed in the texts. The teacher who doesn't employ the precise definitions accepted in previous grades, will certainly confuse students, and may be personally embarrassed. In the matter of definitions a caution should be noted. Although teachers have a responsibility to use exact wording and to be aware of exact meanings, it should not be considered necessary to require pupils to reproduce lengthy definitions. It should also be noted that definitions used in one authorization are not always used in exactly the same way in the other authorization or in other texts. The teacher who is aware of these differences in usage can use them to illustrate the freedom an author has in choosing his definitions.

2. Symbolism

Many of the symbols and notations introduced in Grades VII and VIII were not employed in traditional texts, and may differ slightly from those employed in other modern texts. Pupils will already be familiar with the specific usages of the series, and may not be too tolerant or sympathetic towards a teacher who is not prepared to use accepted notation. Because the new authorizations stress precision of symbolism, the teacher can be confident of consistency only by reviewing the texts of the preceding grades. The symbolism used in each authorization differs in some details from that used in the other textbook series. In view of the common departmental examination it is advisable for the teacher to be familiar with the alternate symbols.

3. Sequential Nature of Development

Because each of the Grade IX authorizations is the third book of a three-book series, the work at the Grade IX level depends heavily on the work of the preceding years. The first introduction of a concept is often designed to provide a general familiarity without intending mastery or complete understanding on the part of the student. The ideas are frequently reinforced by a later treat-

ment with further examples. Eventually mastery is expected and subsequent reviews are inserted to maintain skills and understandings. It is essential that the teacher be aware of the stage of development of a particular concept. Such an awareness can only be obtained by a careful study of not only the Grade IX text but also the texts used in previous grades.

4. Emphasis on Understanding

The authors of both Grade IX authorizations have planned their texts with a view to developing in pupils an understanding and appreciation of the structure of mathematics. Opportunities are provided to stress the recurring characteristics common to various number systems, and to show the repeated applicability of certain principles and properties. The teacher should feel some responsibility to present the materials and to direct the discussion in such a way that the pupils are encouraged to understand and appreciate the generalizations. The teacher who understands and obviously appreciates the beauty of mathematical patterns can do much to help pupils find enjoyment in their mathematical studies.

5. Need for Continuous Drill

The present texts attempt to emphasize understanding, rather than the repetition of procedures. Although the psychology of learning indicates that forgetting is less rapid when there is understanding of materials learned, forgetting does occur. Hence, repeated drill and review must be incorporated into instruction if concepts are to be established firmly. For some students the text may provide ample drill, but teachers must be prepared to supplement or select from the exercises and drills in the text to meet the needs of their pupils.

When a teacher is using a text for the first time there is a natural tendency to move more slowly and deliberately over the unfamiliar portions and to cover more familiar topics hastily. This tendency can result in the teacher unconsciously negelecting to develop adequately those computational skills which were considered a central part of traditional mathematics programs. These skills are still essential. Teachers must be very careful to incorporate into their instruction provision for sufficient practice to develop and maintain computational skills.

Elsewhere in this guide the teacher will find a suggested time allotment for the various units. If difficulty is being encountered in keeping on schedule it should be noted that possible omissions are listed for each text. When it is necessary to gain time, the teacher should omit such optional topics rather than neglect review or drill which is essential. It is recommended that the first time a teacher uses a text only the minimum program be undertaken, making certain that adequate short drills are included. When further time is available optional material may be introduced.

6. Lesson Preparation

The introduction of a new text always demands more time and effort in lesson preparation on the part of the teacher. The demand in respect to the Grade IX authorization is particularly

great for a number of reasons. Effective teaching at this grade level requires a thorough knowledge of the new programs in Grades VII and VIII. Because both series utilize the discovery approach in lesson preparation the teacher must work through a greater bulk of developmental material than was the case in traditional programs. The authors have provided material in teaching guides which should be studied carefully during planning sessions. If gaps and overlaps are to be avoided the spiral method of presenting ideas requires a great measure of long-range planning. The lesson development as presented in the text may be unsuitable for a particular class, thus requiring substantial reorganization on both a short and a long-range basis. Finally, since some of the ideas and approaches may be unfamiliar to the teacher, additional time will be required for the study of the new material itself.

7. Visual and Manipulative Material

The textbook is undoubtedly the most important aid in planning the mathematics program. However, to be interesting and meaningful to students, the program must be supplemented by a careful selection of visual and manipulative aids. These materials should be easily accessible and should be used systematically to help clarify and simplify the mathematical ideas treated in the textbooks.

It is impossible to provide anything like a complete list of visual and manipulative aids. Generally, however, such materials can be divided into the two categories:

1. Commercially prepared material
2. Raw material

Commercial materials designed to exhibit some of the properties of the Hindu-Arabic numeration system, can be used to illustrate structural characteristics of a mathematical system. Prepared geometric models of polyhedra enable the student to identify important properties of solids. Mathematical games and puzzles may help the student to visualize certain spatial relationships more clearly. Such materials are regularly advertised for sale in professional journals. However, since commercial materials are usually prepared for a specific purpose their usefulness is limited. Moreover, their structure is complete and if students are limited to these kinds of materials they miss the opportunity of building structures for themselves.

Priority should be given in the mathematics classroom to the provision of raw materials which will permit the student to produce his own models and visual aids. For example, in the study of polyhedra the student can construct models using wooden rods of various lengths, and modelling clay. It is likely that he will become more familiar with the structure of polyhedra in general than the student who is provided only with ready-made plastic models to handle. An examination of the advertisements in the official journals of the National Council of Teachers of Mathematics will provide the teacher with many practical ideas.

Mathematics classrooms should be equipped with kraft paper, string, peg board, modelling clay, pins, wooden or plastic rods, and other materials which can be used to help students discover mathematical relationships on their own. To get maximum use from such materials there should be regularly scheduled laboratory periods in which students are free to experiment and explore. The main purpose of such periods should not be to produce models for display. The object should be to encourage students to become involved in the discovery and application of mathematical principles and relationships through their own manipulations.

EXPLORING MODERN MATHEMATICS
BOOK 3
GENERAL CONSIDERATIONS

1. Teacher's Edition

Exploring Modern Mathematics, Book 3, Teacher's Edition, is available from either Holt, Rinehart and Winston of Canada, Ltd., or the School Book Branch, Department of Education, Edmonton, as a separate publication. Each teacher of this program is advised to have a copy of the *Teacher's Edition.* The commentary for teachers on pages XI, through XXV, and on pages 501 - 502 of the *Teacher's Edition, Book 3,* is considered to be essential background for the teacher, while the explanations and answers to exercises are placed conveniently for teacher use.

Exploring Modern Mathematics, Book 1, Teacher's Edition, and *Exploring Modern Mathematics, Book 2, Teacher's Edition,* should also be available in order that the teacher may acquire a background in the program.

2. Let's Explore Exercises

By using the **Let's Explore** exercises, the student is encouraged to discover for himself each new concept. THESE EXPLORATORY EXERCISES ARE NOT TO BE GRADED. It is considered essential that the teacher work through the **Let's Explore** exercises in advance of the class to anticipate the students' problems.

The teacher may select **Let's Explore** exercises to suit the abilities of the students. In some instances, short discussions may replace some exercises for superior students. Others may prove to be too difficult for slower students. Teachers who have taught this program have found that in general the exploratory exercises make a useful, stimulating and vital contribution to their teaching. Some of these may require supplementary questions for students who are having difficulty.

Experience has shown that it is useful to spend a short time at the beginning of the class period to create some curiosity about the **Let's Explore** exercises to follow. The class should then work through the exercise independently, the teacher being careful to give assistance only to those students who obviously are making no progress. Students who finish first might be encouraged to do appropriate questions in the practice exercise which usually follows. It is important to bring the class together following the exploratory exercise to discuss the concepts under consideration. On some occasions, it will be necessary for the teacher to work at the blackboard with the entire class, developing the concepts again and reviewing the conclusions.

The assignment of a **Let's Explore** exercise as homework has not been found satisfactory, since experience indicates that individual differences within classes are often great, and the teacher must be available to provide assistance. Teachers have found that most students can do the exploratory exercises in class, and that sufficient understanding can be gained for students to begin the practice exercises which can then be finished as homework.

3. Programmed Supplement

The *Programmed Supplement, Exploring Modern Mathematics, Book 1,* and the *Programmed Supplement, Exploring Modern Mathematics, Book 2,* are excellent aids for the student who has not used this series before Grade IX.

No *Programmed Supplement* is available for *Book 3* at present.

4. Sequence of the Text

Only after very careful thought should the teacher contemplate changing the sequence of the course. It is further suggested that teachers do not omit any section other than those marked "(*)", "Optional" or those suggested in this curriculum guide.

Teachers presenting this course for the first time may find it useful to omit all sections marked "Optional" for average and below average classes, in order to complete the programs.

5. Proofs

Proofs are more rigorous at the Grade IX level than at the Grade VIII level. The teacher must encourage an awareness of the nature of proof. However, experience indicates that some of the proofs in the Grade IX program are difficult. It is likely that the teacher will find it necessary to develop such proofs with students and should use discretion in the extent to which they require students to replicate formal proofs.

6. Applied Problems

The *Teacher's Edition, Book 3,* contains definite comments about the development of problem solving skills for Grade IX students. Pages XVII and XVIII, and the **Overview of Chapter One,** pages XVIII and XIX, should be read with a view to adopting the approach and suggestions made therein.

At the end of each chapter there are exercises of applied problems which are graded as to difficulty.

7. Review Exercises

Teachers are referred to *Exploring Modern Mathematics, Book 3, Teacher's Edition,* pages XVI and XVII for suggestions about the use of review material and applied problems. In particular, note should be made of the frequent use of applied problem and review questions throughout the unit, rather than as a large block assignment at the end of a teaching section.

8. Tests

Two separate publications, *Tests for Exploring Modern Mathematics, Book 3,* and *Key to Tests for Exploring Modern Mathematics, Book 3,* may be ordered directly from the publishers of the series, or through the School Book Branch, Department of Education, Edmonton. Teachers will find these publications an aid to securing an accurate evaluation of the student's progress.

9. Definitions

	Seeing Through Mathematics	Exploring Modern Mathematics
Natural Numbers	0, 1, 2, . . .	1, 2, 3, . . .
Counting Numbers	1, 2, 3, . . .	1, 2, 3, . . .
Whole Numbers	This term is not used	0, 1, 2, . . .

In the *Exploring Modern Mathematics* programs, the counting numbers are defined as the set 1, 2, 3, . . . , and the natural numbers are defined as the same set. The whole numbers are defined as the set 0, 1, 2, ... Note that these definitions are not the same as the definitions used in the *Seeing Through Mathematics* program.

10. Further Comments

Certain information contained in the *Teacher's Edition, Exploring Modern Mathematics, Book 3,* is essential to the proper understanding of the program. The integration of arithmetic, algebra and geometry where feasible, the increased attention to reading, the emphasis on mathematical structure, the development of proofs, and the cautions about forced verbalization of concepts should be noted by the teacher. Pages XI and XII of the *Teacher's Edition* contain paragraphs useful in understanding the nature of this series of textbooks.

MINIMUM PROGRAM, TIME SCHEDULE, OPTIONAL CONTENT

a. Minimum Program

The Grade IX program from *Exploring Modern Mathematics, Book 3,* is as follows:

Chapter 1

Chapter 2

Chapter 3 — Omit Completing The Square, pages 138 through 140.

Chapter 4 — Omit Long Division and Division of Polynomials, pages 178 through 184.

Chapter 5 — Omit Approximate Square Roots and Graphical Approximation, pages 238 through 242.

Chapter 6 — Omit L.C.M. of Polynomials and the remainder of the chapter, pages 283 through 305.

Chapter 7

Chapter 9 — is optional for an interim period and

Chapters 8 and 10 are not part of the Grade IX program.

b. Time Schedule

The following approximate time schedule appears to be suitable for most students.

	Min.	Max.
Chapter 1 — Applied Problems, Conjunction of Equations	4 –	6 weeks
Chapter 2 — Similar Figures	4 –	6 weeks
Chapter 3 — Polynomials in Several Variables	3 –	4 weeks
Chapter 4 — Number Sentences and Proofs	3 –	4 weeks
Chapter 5 — The System of Real Numbers	4 –	6 weeks
Chapter 6 — Fractional Phrases and Equations	4 –	6 weeks
Chapter 7 — Geometry in Three Dimensions	3 –	5 weeks
TOTAL	25 –	37 weeks
Chapter 9 — Graphs, Relations and Functions (OPTIONAL)	3 –	4 weeks
	28 –	41 weeks

c. Optional Content

Content which may be optional for classes or individual students of moderate ability is marked by an asterisk (*) or indicated as optional in the title. Optional sections are not part of the minimum program. Special note should be made of the key used on page 40 of the *Teacher's Edition, Book 3,* classifying problems by level of difficulty. Questions with a circle (●) or a square (■) beside them are classed as of greater-than-average difficulty and of special difficulty, respectively. Such questions may have to be considered optional content for slow classes.

LESSON COMMENTS

CHAPTER 1. APPLIED PROBLEMS, CONJUNCTIONS OF EQUATIONS

Optional Content: None
Omissions: None

A. Problem Solving, pages 1 - 15

The following points appear to be of importance for this topic.

1. A strategy for problem solving is outlined in the display on page 3. Translating problems into mathematical language or number sentences is the most difficult task of problem solving for many students. Adequate time should be used for this topic.
2. On pages 13 and 14, the problems with either insufficient or excess information develop the student's skill in selecting the facts necessary to solve a problem.
3. Although the exploratory method is very effective, students may also learn by observing the teacher present the problem and by stating aloud the steps in the solution process. The teacher should proceed slowly and may write the steps to the solution on the blackboard in order to show the application of the strategy in the display on page 3. An alternative is to have the teacher and class cooperate in solving a problem, pooling ideas and criticisms.
4. Encouraging students to think through a problem appears worthwhile, but the teacher must allow time for this to take place in order to avoid the tendency to memorize type solutions. Students must learn that problem solving methods are not unique. If the teacher tries to avoid too much telling, and permits students to use their own methods, valuable mathematical experience should take place.
5. The teacher must accept some of the students' conclusions worded in their own language. Verbalization should not be forced because students may understand and yet not be able to verbalize meaningfully.
6. Encouraging students to make up their own problems, or to bring to class applied problems from outside the classroom, assists students in better understanding.
7. Problems may be selected to suit the ability of the class. See the key to difficulty, page 40.

B. Conjunctions of Sentences, pages 17 - 37

1. Translating Problems to Conjunctions of Sentences.
   In applied problems, translating to a conjunction of equations is often easier than translating to a single equation. The development of the topic includes:
   (a) Definition of a conjunction of sentences and condition of truth, page 17.

(b) Practice in translating problems to conjunctions of sentences, pages 17 - 19.

(c) A summary of the method of translating problems to conjunctions is shown in the display on page 19.

2. Solving Conjunctions of Equations, page 21 - 37.
"Solving" an equation means finding its solution set.

(a) The Substitution Method, pages 22 - 37.
In "substitution" one name of a number is replaced by another name for that number. This method of solution is used as long as calculations are simple.

(b) The Addition Method, page 27.

i. Review the multiplication and addition principles.

ii. The multiplication principle may be used to produce the additive inverse of one of the algebraic terms. Then by means of the addition principle, one variable is eliminated. Notice that equations are not added together. Identical expressions are added to each side. Students find this method simple and make few mistakes using it.

(c) The teacher's note on page 27 of the *Teacher's Edition* states that the subtraction method is acceptable.

(d) The Check, page 30.

i. The check is important. The conjunction is assumed to be true and an ordered pair is found. However, the converse is required, i.e., the ordered pair that has been found must make the conjunction true.

ii. Relate the check to conditional sentences. Review the topics, **Conditional Sentences,** pages 40 - 43 and **Converses,** pages 43 - 45 and 276 - 277. In *Exploring Modern Mathematics, Book 2.*

e.g.: If $x + y = 4$ and $x - y = 2$, then $x = 3$ and $y = 1$.

Converse: If $x = 3$ and $y = 1$,
then $x + y = 4$ and $x - y = 2$.

(e) Most of the conjunctions have one ordered pair in the solution set. *Exploring Modern Mathematics, Book 2,* page 390, emphasizes the notation of a solution set by writing $\{(a, b)\}$. However, in Book 3, page 30, the authors say that little is to be gained

by writing the solution set in this way and prefer to have students write (a, b) as the solution and not the solution set. Set notation is retained for those conjunctions with no solution. Set notation is not used for those conjunctions with many solutions.

C. Some Special Kinds of Applied Problems, pages 38 - 48.

Review the strategy for solving applied problems, pages 3 and 19.

1. Lever or Torque Problems

(a) The method of solution should be discussed with the science teacher because the topic occurs in the science unit on machines.

(b) Review the law of levers and torques, **Book 2,** pages 252 - 255.

(c) Discuss the convention of representing forces.

(d) Stress the importance of diagrams.

2. Motion Problems

(a) Review the definition of speed $s = \frac{d}{t}$. Using the multiplication principle either t or d may be found.

(b) Encourage students to draw diagrams of the situations.

(c) Work through the problems according to the strategy.

(d) Estimations are valuable.

3. Work Problems

(a) **Let's Explore,** page 56, develops the basic work principle.

(b) Estimate.

D. Reviews in Chapter 1

1. Throughout the chapter, reviews of Grade VIII work are provided. These form a review of background necessary for Chapters 2 and 3. The review exercises at the end of Chapter 1 may be delayed until the class is ready to begin Chapters 2 and 3.
2. Some system of continuous review, perhaps five minutes daily, or at regular intervals, is helpful. Oral drills, as well as written drills, are effective.
3. The review exercises at the end of the chapter may be used throughout the chapter or at a later time.

CHAPTER 2: SIMILAR FIGURES

Omissions: None

Optional Content:

Page 60 Exercises, numbers 12 and 13

Page 73 Exercises, number 7 b

Page 74 Exercises, number 12 a , b

Page 80 Exercises, number 4 a , b

Page 82 Section 2.11, **Let's Explore:** proof that if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a + b}{b} = \frac{c + d}{d}$.
(Omit for very slow classes.)

Page 84 Exercises, number 7 c

Page 93 Pythagorean Property of Right Triangles
The proof as developed in *Book 3,* page 93, and in Section 2.13, **Let's Explore,** page 94, is optional only because the property is included in *Book 2*. The proof developed in *Book 3* is different from the proof given in *Book 2*.

In this chapter the concept of similarity of triangles is developed. It is then used in applied problems, as well as applied to other mathematical concepts.

A. Congruent Triangles, pages 55 - 60

1. Review the definition of plane figures, *Exploring Modern Mathematics, Book 2,* page 59.
2. Review congruent figures, *Book 2,* page 63.
3. With these ideas start to study congruent triangles. In congruent triangles there is a one-to-one correspondence between the vertices and a similar correspondence between the sides. An agreement about notation showing this correspondence is adopted. Notice the paragraph above the display on page 57. A slow class may cut out triangles, place them together to find the corresponding vertices and angles, and the properties of congruent triangles.
4. Review the construction of triangles, *Exploring Modern Mathematics, Book 2,* page 73, so that congruent triangles may be constructed.

B. Similar Figures, pages 61 - 86

1. **Let's Explore,** page 61, provides the background which leads to the definition of similar figures. Some discussion of examples of similar figures such as maps, scale drawings, projection of pictures, dress patterns and models, is desirable.

2. Similar Triangles, pages 62 - 66

   (a) **Let's Explore** 2.3, page 62, provides experience which should enable students to define similar triangles. Be sure students can read the symbol $\sim$. Compare the definitions of similar triangles and congruent triangles.

3. Conditions for Similarity, pages 66 - 75

   (a) Review the sum of the measures of the angles of a triangle.

   (b) The **Let's Explore** exercises in this section provide material for students to discover for themselves the conditions for similarity. Slower students will require help. Students may make charts of the conditions for similarity. Drill on the conditions of similarity is necessary.

4. More about Ratios, pages 75 - 80

   (a) In this section the ratio of two sides of a triangle is compared to the ratio of the corresponding two sides of a similar triangle. **Let's Explore** 2.9, page 78, establishes that properties of ratios and proportions do not depend on triangles but are properties of numbers.

   (b) Proportion, pages 81 - 84

      i. The concept of proportionality is very difficult to explain without considerable preparation.

      ii. Definitions of proportion and proportional.
      Be sure students can read $a/b = c/d$. Slower students may have difficulty with these concepts. Students may illustrate the patterns of ratios with examples of their own. They should also prove the converses of the displays on pages 82 and 83.

C. Direct Variation, pages 86 - 90

1. Review ordered pairs.
2. For a discussion choose ordered pairs such as (1,7), (2,14), (3,21). Let the students find the pattern of the ratios and then write $y/x = 7/1$, and finally, by the multiplication principle, $y = 7x$.
3. Derive definitions for direct variation and the variation constant.
4. Relate direct variation to formulas in mathematics and science.

D. Graphs

1. Review graphs of ordered pairs, *Exploring Modern Mathematics, Book 2,* pages 200 - 204.
2. Graph $y = kx$. The graph of an equation of direct variation consists of points on a line because rational numbers are used.
3. By means of similar triangles it is proven that $y = kx$ passes through the origin.

E. Pythagorean Property of Right Triangles, pages 94 - 98

1. The proof, based on similar triangles, is difficult. A diagram should be placed on the blackboard and through discussion the corresponding sides and angles located. Colored chalk may be used to mark the corresponding parts. Slow students need learn only the display, page 97, and its application to problems.
2. Students should look for practical examples of right triangles such as rafters, ramps, etc.
3. For the problems on page 98, the square root tables on page 479 may be used. However, students may need to know a formal method of calculating square roots for some problems. The square root tables in the text are not adequate for all problems, e.g. page 98, number 3 d, 7, 8.

F. Indirect Measurement

1. Review the properties of similar triangles and right triangles, and stress the location of corresponding sides and angles.
2. Review the Pythagorean property for right triangles.
3. Review the strategy, page 3.
4. Stress the importance of diagrams.
5. Have students make estimations.

G. Applied Problems and the Review

These should not be left until the end of the chapter. Use them throughout the program for review.

CHAPTER 3: POLYNOMIALS IN SEVERAL VARIABLES

Omissions: Completing The Square, pages 138 - 140.

Optional Content: None

This chapter closely parallels *Exploring Modern Mathematics, Book 2,* Chapter 8. Polynomials in one variable were considered there, but in this chapter polynomials in more than one variable are considered. Thus, the earlier concept is extended, and at the same time is reviewed.

A. Polynomials in Several Variables, pages 111 - 115

1. Review the terms, "degree of a term", "degree of a polynomial" and "coefficient". It is not important that the student be able to define these terms, but the use and understanding of them is considered essential.

2. Similar terms, pages 113 - 114
   (a) Review equivalent phrases.
   (b) Define similar terms.
   (c) Combining similar terms is based on the distributive property of rational numbers.
   (d) Arrange terms in ascending or descending powers of one of the variables.

B. Addition, pages 115 - 116

Review the properties of addition, *Exploring Modern Mathematics, Book 2,* page 328. The distributive property for rational numbers is the basis for adding. Students should show how this property is applied in some of the questions.

C. Additive Inverses and Subtraction

Review the definitions of additive inverses and subtractions, *Exploring Modern Mathematics, Book 2,* pages 330 - 331.

D. Multiplication, pages 118 - 138

Review the properties of multiplication for polynomials, *Exploring Modern Mathematics, Book 2,* page 341, and extend the properties to polynomials in more than one variable. Review the properties of exponents.

1. Types of multiplication
   - (a) Two monomials, page 118. Watch the use of parentheses by students.
   - (b) Monomials and binomials, page 119. Arrows may be used to indicate the distributive property.
   - (c) Two polynomials.
   - (d) Special Products of Polynomials, pages 120 - 122. The special products patterns from *Exploring Modern Mathematics, Book 2,* pages 342 - 348, should be reviewed and a chart made.
     - i. Product of any two binomials.
       Show how the distributive property for rational numbers applies.
     - ii. Square of a binomial.
     - iii. Product of the sum and difference of two phrases.
       Students must learn the patterns. Do as much oral work as possible.

2. Factoring, pages 123 - 138

   Factoring is the reverse of multiplication. If the multiplication patterns have been thoroughly learned, certain types of factoring present no difficulty. Give drill on recognizing patterns.
   - (a) Common factor, page 123.
   - (b) Difference of two squares, page 125.
     Note the importance of the highest common factor.
   - (c) Trinomials which are squares, page 127.
     Review the pattern for squaring a binomial. Look for two terms which are squares.
   - (d) Trinomials which are not squares, page 130.
     This is the most difficult type of factoring. For slower students, drill on factors of numbers and sums of those factors.

3. Rules for Factoring, pages 131 - 137
   - (a) If necessary, arrange the terms in ascending or descending order of a variable.

(b) Look for a common factor. If there is a common factor, remove it.

(c) Examine the exponents. If the exponents of the first and third terms are even numbers, and the exponents of the middle term are half those of the first and last term, the polynomial may be factorable. The factoring should be checked. A great deal of drill is necessary for speed and accuracy.

E. Applied Problems

1. The mixture problems are difficult for many students. Try to find two descriptions for the amount of solute. This is easier than finding two descriptions of the solvent.
2. Are you using the review problems throughout the chapter?

CHAPTER 4: NUMBER SENTENCES AND PROOFS

Omissions: Long Division and Division of Polynomials, pages 178 - 184.

Optional Content: None

Students have had experience with informal proofs in equations in Grade Eight, and consequently, will not experience a great deal of difficulty. In formal proofs the students number the assertions and write the reasons to support them.

A. Writing Proofs, pages 151 - 155

1. Assertions — things we say in a proof.
2. Hypothesis — something we assume or suppose to be true.
3. Review the number properties and principles on page 154 and have the students give examples of each.
4. At first students might solve the equations using the equation-solving principles as they usually do. Then discuss with them the properties of numbers or principles used and write a formal proof. Good students may combine some of the steps. Encourage slower students to write a rough draft.
5. Proofs are not unique. However, the relative merits of proofs should be discussed.
6. Compare the conclusion of the proof with $A \rightarrow B$. $A \rightarrow B$ is true if the solution set of A is a subset of the solution set of B.
7. From the proof it is not known whether the solution set has been found or not. Consequently it is necessary to prove the converse of the statement just proven.

B. Solution Sets and Truth Tables, pages 156 - 164

1. Conditional Sentences, page 156
   (a) Review the conditional sentence, converses, truth tables, consequent, antecedent and subset, and Venn diagrams, *Exploring Modern Mathematics, Book 2,* pages 40 - 45.
   (b) Encourage the students to make up examples of conditional sentences and find the truth values.

2. Conjunctions, page 157

   (a) Review conjunctions of equations, page 17.

   (b) Draw Venn diagrams "and—intersection".

   (c) Truth tables are used for conjunctions.

   (d) Students may make examples.

3. Disjunctions, page 159

   (a) Definition.

   (b) Venn diagrams "or – union" are developed.

   (c) Truth tables are used for disjunctions.

   (d) Encourage student examples.

Note: The diagram, page 159, in the text does not illustrate $x < 2$ or $x = 5$ (system of whole numbers.) The Venn diagram for the example should be as shown below. The Venn diagram in the text is for a general situation.

4. Negations, page 160

   (a) Review negations, *Exploring Modern Mathematics, Book 2,* pages 32 - 33.

   (b) In negating a mathematical sentence, a sentence is formed whose solution set is the complement of the original sentence.

   (c) Truth tables are used for negations.

   (d) Venn diagrams are used for negations.

C. Equivalent Sentences, pages 164 - 170

1. Sentences A and B are equivalent if they have the same solution set. Differentiate between equivalent sentences and equivalent expressions. Expressions are equivalent when they name the same number for every replacement that is sensible for both of them. To prove A and B are equivalent students prove $A \rightarrow B$ and $B \rightarrow A$. Be sure students can read and explain $A \leftarrow\rightarrow B$.

2. Contrapositives, pages 166 - 170

   (a) Review the conditional sentence, its converse and negation. Show how the contrapositive sentence and conditional sentence are related.

   (b) Truth tables are used for contrapositives.

   (c) Students should make up examples.

D. Inequalities, pages 171 - 177

1. Solving inequalities

   (a) After solving equations and writing proofs, students find solving inequalities rather easy since they use the same

properties of numbers and the addition principles as for equations. The idea is to find a sentence so simple, e.g. $x > 5$, that its solution set is obvious.

(b) The form of the answer is rule notation, e.g. $\{y \mid y > 6\}$, for infinite sets and roster notation, e.g. $\{3, 6, 9\}$, for finite sets.

(c) The check consists of proving the converse by reversing the steps of the solution.

(d) The multiplication principle of inequalities, page 175, is different from the one for equations and is more complicated. **Let's Explore,** page 174, handles this and students should be able to generalize the rules. Discuss the display, page 175.

E. Equivalent Equations, pages 185 - 190

1. Equivalent equations are a special case of equivalent sentences, page 164, having the same solution sets.

2. If an equation is expressed in a different form by applying one of the principles, the equations are equivalent.

   (a) Prove the converse of the addition principle
   $a = b \rightarrow a + c = b + c$
   Students should be able to read $a = b \longleftrightarrow a + c = b + c$

   (b) Prove the converse of the multiplication principle.
   If $c \neq 0$, $a = b \longleftrightarrow ac = bc$
   Good students will enjoy the fallacious proof for $2 = 1$.

$$\begin{aligned} a &= b \\ a^2 &= ab \\ a^2 - b^2 &= ab - b^2 \\ (a - b)(a + b) &= b(a - b) \\ a + b &= b \\ 2b &= b \\ 2 &= 1 \end{aligned}$$

F. Equivalent Inequalities, pages 190 - 192

This section parallels the section on equivalent equations. To prove that two inequalities are equivalent, students should:

either—prove the converse of the addition principles for inequalities,

$$a > b \rightarrow a + c > b + c$$

or — prove the converse of the multiplication principles:

for any positive g, $a < b \rightarrow ag < bg$, $(g \neq 0)$,

for any negative g, $a < b \rightarrow ag > bg$, $(g \neq 0)$.

G. Applied Problems

Review the strategy for solving problems, page 3 and page 19.

CHAPTER 5: THE SYSTEM OF REAL NUMBERS

Omissions: Square roots and graphical approximations, pages 238 - 242

Optional Content: None

A. Introduction

The theorem of Pythagoras is used to introduce real numbers. A right triangle with legs one inch long is drawn. Students try to find a rational number whose square is 2 and discover that the square root of two is not a rational number.

B. Factoring Squares of Natural Numbers, pages 203 - 205

1. This topic introduces the proof that square roots of natural numbers that are not perfect squares are not rational numbers.
2. Review prime factors, *Exploring Modern Mathematics, Book 1,* Chapter 4.
3. Students factor the squares of natural numbers into prime factorizations and examine the exponents. From this they conclude that if a natural number is a square, every exponent is even and the square root is a rational number.

C. Rational Numbers Without Square Roots, pages 206 - 208

Some superior students will be able to work through this section on their own. However, all students will profit by a teacher-led discussion. Slow students should omit the proof but should know the generalization that prime numbers have no rational square roots.

D. Gaps in the Number Line, pages 209 - 211

1. Students now realize that since the square roots of prime numbers and the square roots of numbers that are not perfect squares are not rational numbers, these roots will not appear on the number line. Hence, we need a number system which will include them.

E. Real Numbers, pages 211 - 224

1. Discuss the composition of each of the number systems studied thus far.
2. The new number system, the set of real numbers, contains all of the rational numbers as well as the new numbers we call irrationals. Each point on the number line now has a corresponding number.
3. Discuss the placing of the points on the number line to correspond with irrational numbers, page 211.
   (a) Some irrational numbers are square roots, e.g. $\sqrt{2}$.
   (b) Many irrationals are not square roots and are given a letter name, such as π.
   (c) Teacher's note page 214, Exercise 4, states that any unending, non-repeating decimal represents an irrational number.

4. Additive inverse signs may be written as raised minus signs or written at the middle of the line.

5. Properties of Real Numbers, page 214.
From charts of number systems show the similarity between the rational number properties and the real number properties. Only one new property is introduced in the system of real numbers.

6. Absolute Values of Real Numbers, page 216.

   (a) Review absolute value, *Exploring Modern Mathematics, Book 2,* page 165. Absolute value for real numbers is defined in the same way as it is for rational numbers.

7. Exercises 6 and 7, page 217, show that the set of irrational numbers does not have the closure properties for addition and multiplication.

8. Theorems about Real Numbers, pages 219 - 224.
If time permits, superior students should be encouraged to do some of the proofs.

F. Polynomials with Real Number Coefficients, pages 225 - 231

This section is an extension to real numbers of the content in Chapter 3.

1. Real numbers have the same basic properties as rational numbers. Addition, subtraction and multiplication are done as in the other systems studied, but simplification is not done as readily. When a rational approximation to an irrational number is needed while working a problem, it is best to wait as long as possible in the solution method before introducing such an approximation.

2. Factoring, page 226

   (a) A difference of two squares is factored the same as before.

   (b) Completing the square should be omitted, because completing the square on page 138 has been omitted.

3. A Convenient Notation, pages 228 - 231

   (a) The notation, P(x), refers to polynomials in one variable and is an introduction to function notation.

   (b) Students may think of the new notation, P(x), which is read 'P of x', as a shorter way to write about polynomials in general.

G. Graphs on a Plane, pages 231 - 137

1. Graph of $y = kx$, page 232

   (a) Review the plotting of ordered pairs of rational numbers. Discuss why graphs from ordered pairs of rational numbers are not lines. Discuss why graphs from ordered pairs of real numbers are lines.

(b) Review the direct variation $y = kx$, page 86. By discussion with slower students conclude that:

i. if k is positive, the graph slants upward to the right.

ii. if $k = 0$, the graph has zero slope and is the x– axis.

iii. if k is negative, the graph slopes downward to the right. Students require considerable experience in graphing equations.

2. Graphs of $y = mx + b$, page 233

(a) Discuss equations such as $y = 3x$ compared to $y = 3x - 5$. From graphs show $y = 3x$ is 5 units "above" $y = 3x - 5$ and 5 units "below" $y = 3x + 5$. This concept is not difficult but requires drill. Encourage neat work.

(b) Discuss the generalized forms of the above equations, $y = mx$, with reference to the display on page 234.

(c) Students should recognize that the conjunction of equations such as $y = mx$ and $y = mx + b$ have no solution. (See page 27, problems 29 and 30). Now, they should realize that those conjunctions are the equations of parallel lines. It would be a good idea to show by graphs conjunctions with only one solution. Other graphs could show conjunctions with many solutions.

(d) Point out that every first degree equation has a graph which is a straight line.

3. Graphs of Quadratic Functions, page 237

(a) Students will be able to complete page 237 with no difficulty.

(b) Graphs of quadratic equations are considered in Chapter 9 (optional).

H. Equations and Inequalities, pages 243 - 251

1. The same principles and properties used for solving equations and inequalities in rational numbers apply to real numbers. The only difference is that it may not be possible to simplify the answer as well as when we work with real numbers.

2. Formulas, pages 245 - 250.

(a) The importance of formulas and their manipulation by engineers and scientists should be discussed. Some examples from the science course could be discussed. Drill on the manipulation of formulas is needed.

(b) Students should be encouraged to bring formulas from their area of interest.

I. Applied Problems, pages 251 - 254

1. Angles and Rotations.
This can be treated as a reading exercise for top students. Others will profit from a discussion of angles and rotations.

2. Review problems solving procedures, pages 3 and 19.

CHAPTER 6: FRACTIONAL PHRASES AND EQUATIONS

Omissions: L.C.M. of Polynomials and the remainder of the chapter, pages 283 - 305

Optional Content:

Page 264 Exercises, number 13. The teacher's notes in the *Teacher's Edition, Book 3,* suggest that this may be omitted for some students.

Page 276 Exercises: The listing of non-sensible replacements as in the directions should be considered optional. Some of the cases are a little involved.

A. Introduction, pages 261 - 264

This chapter introduces the student to the important class of fractional expressions of the form $P_1(x)/P_2(x)$, (where $P_2(x)$ is not the zero polynomial.)

The student should recall that any number phrase must be able to name a number for some replacement of its variable, or already be the name of a number (page 35, *Exploring Modern Mathematics, Book 2*). This rules out any replacement for x which makes $P_2(x) = 0$. He needs also to recall the notations $P(x)$ and $P(a)$ which was introduced on pages 228 - 230.

The teacher's advisory notes on page 262 explain why the text does not develop a formal mathematical system for these fractional expressions as was done for polynomials. Unless the students show a desire to explore the reasons further, only casual mention need be made of this difference in treatment.

B. Equivalent Phrases, pages 265 - 283

1. The **Let's Explore** exercises in Equivalent Phrases lead to Theorem 1 which gives a simple test for equivalence of two fractional pharses.

1. Multiplication

Since $\frac{P_1(x)}{P_2(x)} \cdot \frac{P_3(x)}{P_4(x)} \equiv \frac{P_1(x) \cdot P_3(x)}{P_2(x) \cdot P_4(x)}$ the student sees that he can "multiply" fractional phrases in a manner analogous to the way in which he multiplied rational numbers. (Indeed, our fractional phrases are often called rational expressions.) An advantage of the second form over the first is, of course, that it may be possible to simplify the second form. The method of simplification, page 271, is analogous to simplification with rational numbers.

3. Interesting variations on the theme of "Number Tricks", page 273, may be found in almost any text on recreational mathematics.

4. Division

The treatment for "division" parallels that which has been done with rational numbers and is quite straightforward.

5. Addition

The "addition" of phrases of the form $\frac{P_1(x)}{P_2(x)} + \frac{P_3(x)}{P_2(x)}$, page 278, parallels that which has been done with rational numbers. Note, however, that the more difficult case $\frac{P_1(x)}{P_2(x)} + \frac{P_3(x)}{P_4(x)}$ requires a treatment of least common denominator and is omitted from this course.

6. If you feel that omission of "subtraction" is serious, it would be a simple matter to follow the examples on addition with a few examples of the form $\frac{P_1(x)}{P_2(x)} - \frac{P_3(x)}{P_2(x)}$. This need not be time consuming.

7. The chapter provides useful incidental review of items from several topics such as polynomials, factoring, properties of rational and real numbers, equivalent phrases, and others.

CHAPTER 7: GEOMETRY IN THREE DIMENSIONS

Omissions: None

Optional Content:

Page 322 Exercises, numbers 9, 10

Page 338 Exercises, number 3 c

Page 349 Exercises, number 6

Pages 352 - 353 Comparison of Volumes, and 7.14 **Let's Explore**

A. Introduction

This chapter extends to three dimensions many of the ideas previously encountered in the two-dimensional situation. For example, the polyhedron is the three-dimensional analogue of the polygon. The standard geometric figures — the polyhedra, cylinders, prisms, cones, pyramids, and the sphere — are developed together with formulas for area and volume.

B. Topical Outline

1. Three Dimensional Figures, pages 311 - 313
2. Intersections in Space, pages 315 - 321
3. Lines Perpendicular to a Plane, page 319
4. Polyhedrons, pages 322 - 325
5. Cylinders and Prisms, pages 326 - 335
   (a) Areas of Cylinders and Prisms, page 331
6. Cones and Pyramids, pages 336 - 341
   (a) Areas of Cones and Pyramids, page 339

7. Spheres, page 342
   (a) Area of a Sphere, page 342
8. Volume, pages 343 - 351
   (a) Volumes of Cylinders and Prisms, page 346
   (b) Volumes of Cones and Pyramids, page 349
   (c) Volume of a Sphere, page 350
9. Exercises, pages 353 - 357
10. Definitions

D. Definitions

It will be noted that the definition of some space figures, (the cone, page 336, for example), differs from that found in some texts. The teacher's advisory notes and the overview of this chapter give reasons for preferring the definitions found in this text.

E. Useful Suggestions

1. The text places emphasis on student drawing of geometric figures. A serious attempt is made to develop perception in three dimensions and to develop skill in the representation of three-dimensional figures by two-dimensional drawings. Joint participation of students and teachers in sketching three-dimensional mathematical objects, with the teachers using the overhead projector, could be useful. The construction of three-dimensional models is a useful exercise and suggestions are found in the text as well as in such publications as Magnus J. Wenninger's *Polyhedron Models for the Classroom,* which is available from the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D.C. 20036, U.S.A., for 60¢.
2. It needs to be emphasized that the student learns from his **participation** in drawing and in model-making rather than from simply viewing completed drawings and manufactured models. Thus, the excellent projectuals and models available are useful, but they must supplement, rather than replace, student participation.
3. For a mathematics club, a good follow-up to Euler's Formula (problem 10, page 325) is the **Three Houses and Three Walls Problem.** A discussion of this problem may be found in Moses Richardson's *Fundamentals of Mathematics,* revised edition, Macmillan, New York, 1958. Model-making is also a good activity for a mathematics club.
4. Note that the geometric figures discussed are two-dimensional sets of points but many determine a three-dimensional region. We associate area with the former and volume with the latter. Thus, it is proper to talk of the **area of a sphere** and **the volume of a spherical region.**
5. Again, the concept of "sweeps" is used to make plausible certain formulas for area and for volume. Other formulas for area (such as that of the area of the sphere) are too difficult to develop at this level and must simply be presented to the student. The area and volume formulas will be important in later work in mathematics and should be memorized.

CHAPTER 9: GRAPHS, RELATIONS AND FUNCTIONS

Omissions: None

THIS CHAPTER IS OPTIONAL FOR AN INTERIM PERIOD. Further, the material involved is treated in some detail in the present senior high school mathematics program. Nevertheless, teachers of faster groups may wish to use some, or all, of the material of this chapter. The main topics are given in the chapter title, namely, graphs, relations, and functions.

The work on graphs is an extension of the work undertaken in Chapter 5, with emphasis on its concept of **slope.** A third method of solving conjunctions of equations is introduced — that of graphing.

The concepts of **relations** and **functions** lie at the core of mathematics and are important bases upon which much of what follows in mathematics will be built. The concepts of **sets** and **ordered pairs** are basic in leading to the formal definition of relation on page 423. This set-theoretic approach to relations and functions is a very common one today but differs considerably from the treatment found in older secondary school mathematics texts. The examples on page 422 connect the formal listing of the ordered pairs of the relation with the more general verbal statement. These exercises may be supplemented by others involving relations such as "is older than," "is an ancestor of", etc. Students may be encouraged to think of more relations on their own, define the sets A and B and then list the ordered pairs in the relation. **Domain** and **range** are important technical terms.

Graphing relations is a very useful way of illuminating some of the basic ideas, and several examples and exercises are found in the text.

Functions are then introduced as relations of a special kind and again graphs support the main ideas.

Many teachers will have to get rid of old definitions and old notation in order to be consistent with the text. Note that a function is a set of ordered pairs with certain properties. A function may be denoted by a single letter, such as f [**not** by f(x)]; f names a set of ordered pairs. If x is a variable representing the first coordinate, then f(x) is the corresponding second coordinate. The set of all (x, f(x)) where x may be replaced by any number in the domain **is** the function f. f(5) is the second component of that ordered pair whose first component is 5. Equations such as $f(x) = x^2 + 6$ describe the function in the sense that, knowing the domain of x, we can then generate all the ordered pairs of the function. But $f(x) = 2x + 3$ **is not** the function, it is the set of ordered pairs generated that is the function.

Polynomials have been studied previously and the functions that are described by polynomials — the polynomial functions — constitute a very important class of functions in mathematics. The quadratic functions is a simple example of a polynomial function and graphs of quadratic functions are studied. In this chapter we find a good example of how geometric and algebraic ideas may be integrated so that each strengthens the other.

SEEING THROUGH MATHEMATIC, BOOK 3, SPECIAL EDITION

GENERAL CONSIDERATIONS

To parallel the pupil textbook, *Seeing Through Mathematics, Book 3, Special Edition,* the publishers have prepared a *Teaching Guide, Seeing Through Mathematics, Book 3, Special Edition.* It is available directly from:

School Book Branch		W. J. Gage Ltd.
Department of Education	OR	1500 Birchmount Road
Edmonton, Alberta		Scarborough 4, Ontario

No teacher should attempt to teach the course without having the *Teaching Guide* available for constant reference, during the day-to-day preparation. In the *Teaching Guide* the overall purpose of each lesson is outlined, background concepts are presented and teaching suggestions are developed. An early section of the *Teaching Guide* entitled "Introductory Remarks" provides background information and an overview of the course so pertinent that the teacher would be well-advised to examine the total section before starting to teach the first lesson.

1. Semi-Programmed Nature

   A *Response Verification Book* is available from the publishers to accompany *Seeing Through Mathematics, Book 3, Special Edition.* Expected replies are given to all the questions asked in the lesson development, and to the exercises which occur at the end of each lesson. These answers are accompanied by explanatory comment where necessary. Each teacher should have a copy of the *Response Verification Book* available for use in the preparation of daily lessons. For many lessons it is possible for the teachers to handle the developmental questions and portions of the **On Your Own** exercises through an oral approach. Such a technique generally permits more rapid coverage of lessons and increases pupil interest, but requires that the teacher be prepared to deal effectively with pupil responses. The *Response Verification Book* helps the teacher anticipate pupil answers, both correct and erroneous.

   Teachers may desire to make copies of the *Response Verification Book* available for student use. Some lessons might be handled by students themselves using the *Response Verification Book* to check answers. Pupils who have been absent or pupils who have had difficulty with a particular lesson, will find this book most helpful.

2. Minimum Program and Schedule of Units

   The amount of material that can be covered will depend on the ability and mathematical background of the class. The authorized textbook would appear to have sufficient material to meet the needs of the most capable groups. For average and below average classes, teachers would be well advised to attempt only the minimum program outlined below. If time is available the teacher may choose to supplement this minimum program by considering any of the material labelled **Content Not Included in the Authorized Minimum Program.**

SUGGESTED TIME SCHEDULE FOR MINIMUM PROGRAM USING S.T.M. 3

All references to lesson, page, exercise and display numbers refer to the *Special Edition.*

Unit Number	Unit Title	Total Lessons in Text	Lessons in Minimum Program	Teaching Time Required to Cover Minimum Program Including Testing
13	Formulas	10	7	3 weeks
15	Plane and Space Geometry	14	13	7 weeks
16	Vectors	12	2	1 week
17	The Real Number System	10	8	4 weeks
18	Logic and the Nature of Proof	11	6	2 weeks
19	Algebraic Expressions	18	15	12 weeks
20	Equivalent Conditions in One Variable	11	7	3 weeks
21	Relations and Functions	7	0	0
22	Linear Functions and Systems of Conditions	4	0	0
			58 lessons	32 weeks

The authorized minimum program for Grade IX consists of all the material included in the above units of the text except the material listed below.

CONTENT NOT INCLUDED IN THE AUTHORIZED PROGRAM

Note: It is suggested that the teacher clearly mark the text to indicate the portions which are not included in the minimum program.

(a) Special Challenges
Throughout the book the exercises entitled **Special Challenges** are intended for enrichment, and are **not** included in the minimum program.

(b) Omissions by Unit
Unit 13 — Formulas (Note — this Unit was included in *Book 2, Special Edition,* but was not included in the minimum program for Grade VIII, so will not have been covered with most classes. It is required that all students cover this unit.)

Lesson 171 — Relations determined by $y = \frac{k}{x^2}$ (Complete lesson, pages 16 - 17).

Lesson 172 — Display 4, page 21, and Exercise I through 0, pages 20 - 21.

Lesson 173 — Problems involving conditions for variation of the type $y = \frac{k}{x^2}$; display 8 and last two paragraphs, page 27; Exercises 0 through W, page 28.

Lesson 174 — Trigonometric formulas: the last paragraph, page 32; Display 5 and 6; Exercises A through L, page 33.

Lesson 175 — Using formulas involving trigonometric rate pairs to solve problems; Exercise A, page 36 and the paragraph preceding this Exercise; Display 6 and Exercise B through K on page 27.

Unit 15 — Plane and Space Geometry

Lesson 186 — Proofs involving congruent triangles (entire lesson).

Lesson 187 — A proof of the Pythagorean property: Display 3, the paragraph following Exercise N, and Exercise A through E on page 64: page 65; the first paragraph on page 66.

Unit 16 — Vectors: All the material in this unit may be **omitted,** except the two lessons listed below:

Lesson 205 — Addition and Subtraction of Rational Numbers, page 157,

Lesson 206 — and Multiplication and Division of Rational Numbers, page 161.

Unit 17 — The Real Number System

Lesson 213 — A finite field (entire lesson).

Lesson 214 — Proofs of theorems of real numbers: Displays 3, 4, 5, 6, and 7, and all exercises and paragraphs pertaining to proofs given in these displays; **On Your Own** exercises through 6, 8 and 9.

Lesson 216 — Proofs of order theorems of real numbers: Displays 1, 2, 3, 5 and 6, and all exercises and paragraphs pertaining to the proofs given in these displays: **On Your Own** exercises 11 and 13.

Lesson 217 — Examples of indirect proof (entire lesson).

Unit 18 — Logic and the Nature of Proof

Lesson 226 — Proof (entire lesson).

Lesson 228 — Indirect proof (entire lesson).

Lesson 229 — The nature of a mathematical system (entire lesson).

Lesson 230 — Proofs of theorems of real numbers (entire lesson).

Lesson 231 — A geometric system (entire lesson).

Unit 19 — Algebraic Expression

Lesson 235 — A proof of the distributive property for the sum of three or more terms; Display 7, paragraph preceding Exercise I, and Exercise I through L on page 309.

A proof that multiplication distributes over successive subtractions: Display 10, the paragraph preceding Exercise L and Exercises L through 0 on page 310.

Lesson 239 — Development of method for factoring difference of two squares: Display 2 and Exercises A through E on page 322 that pertain to the display.

Lesson 242 — A comparison of integers and polynomials (entire lesson).

Lesson 244 — Proofs of properties for reduction of quotients: Displays 2, 3, and 6, and all exercises and paragraphs pertaining to the development of these properties.

Lesson 245 — A proof of the product property of quotients: Display 1, Exercises C through F on page 347.

A proof of the reciprocal property of quotients: Display 4, the paragraph preceding Exercises A, and Exercises A, B, and C on page 348. A proof of the quotient property of integral powers: Display 7, the paragraph preceding Exercise A, and Exercises A through E, on pages 349 and 350. A proof of the power property of quotients: Display 8, Exercises L through P on page 350.

Lesson 246 — A proof of the sum property of quotients of real numbers: Display 1, Exercises C, D, and E, on pages 351 and 352.

Lesson 247 — A proof of the property that the square root of a product is equal to the product of the square roots: Display 3, paragraph preceding Exercise A, and Exercises A through E in column 1 on page 360.

Lesson 248 — Principal roots and rational powers (entire lesson).

Lesson 249 — Algebraic expressions (entire lesson).

Unit 20 — Equivalent Conditions in One Variable

Lesson 250 — Use of the biconditional to determine the solution set of a condition: Displays 9, 10, 11, 12, 13 and 16, the paragraph preceding Exercise R on page 379 and the remainder of page 379, the paragraphs preceding Exercise K on page 380, **On Your Own** Exercise 5 through 12.

Lesson 251 — Proof of a theorem concerning the solutions of linear conditions for equality: Exercises I through M on page 382 and Display 4 on page 383.

Lesson 252 — Use of the biconditional to find the solution set of a linear condition for equality: Displays 4 and 5, the paragraphs preceding Exercise A on page 386, Exercises A through F on page 386, all of page 387, page 388 through Exercise Y (except Displays 6 and 7).

Lesson 255 — Conditions for inequality that involve the idea of absolute value: the paragraph preceding Exercise A on page 398 and the rest of the development Exercises: **On Your Own** Exercises 1, 2, 4, 7, 8, 9, 10, 12, 13, 14, 23 through 30, 32, 33, 34.

Lesson 256 — Problems requiring conditions for inequality that involve absolute value: Display 8 and Exercises pertaining to it: **On Your Own,** Problem 12.

NOTE: — The units listed below are **not** included in the minimum program for an interim period.

Unit 21 — Relations and Functions

Unit 22 — Systems of Conditions

Unit 13 — Formulas

Unit 13 brings together a number of the mathematical ideas studied to date and applies them to problem solving. The students develop, learn, and apply a number of formulas used in business, science and geometry. Note that each of the Lessons 171 - 175 have some portions not included in the minimum program.

Lesson 168 — Relations Determined by $y = kx$

This lesson contains at least three concepts that are essential to the rest of the work of the unit. The teacher should be quite sure that pupils grasp the meanings of the terms, "direct variation", "constant of variations", and "locus of a solution set."

Notice that Exercises 6 to 10, inclusive, may be omitted except for good students. Since this lesson is basic to those that follow, it is suggested that two class periods be set aside for its development.

Lesson 169 — Relations Determined by $y = kx^2$

Before presenting this lesson the teacher should be acquainted with the excellent ideas found in the *Teaching Guide,* pages 47 - 49. The exercises in the book do not ask the students to draw graphs, but experience has shown that the lesson is ineffective unless the students are required to draw them. Extra exercises provided by the teacher are necessary to give the students practice in drawing parabolas.

Lesson 170 — Relations Determined by $y = \frac{k}{x}$

In this lesson students study conditions for inverse variation. It is suggested that teachers prepare a list of practical situations involving inverse variation. Students can assist in this undertaking as an introduction to the lesson. Notice that conditions for inverse variation can be stated in the form $xy = k$, provided k is not replaced by zero. The conditions $xy = 0$ and $y = \frac{0}{x}$ do not have identical solution sets. Read pages 49 to 51 of the *Teaching Guide* carefully.

Lesson 171 — Relations Determined by $y = \frac{k}{x^2}$

This lesson is not included in the minimum program. It supplies excellent background for better students if time permits its inclusion in their program.

Lesson 172 — Relations Determined by Conditions for Variation.

This lesson is a summary of the three previous lessons which involve the relations determined by $y = kx$, $y = kx^2$, and $y = \frac{k}{x}$. It may be advisable to have the students begin their replacements of x with a negative number.

Example:

$y = \frac{-2}{x}$

x	–3	–2	–1	1	2	3
y	$\frac{2}{3}$	1	2	–2	–1	$\frac{-2}{3}$

Students have a tendency to draw their graphs in one quadrant only if they are not required to begin their replacements in the chart with a negative value. Use the terms, "line graph", "hyperbola", and "parabola" as often as possible, because these words are used again in Unit 17, lessons 218, 219, and 220. Extra exercises may be necessary.

Lesson 173 — Problems Involving Conditions for Variation

Note that the assumption is made that all measures given in the problems are mathematical measures, not physical measures. Problems involving conditions for variation of the type $y = \frac{k}{x^2}$ are omitted from the minimum program.

Lesson 174 — Formulas Derived from $\frac{y}{x} \sim \frac{k}{1}$

In this lesson students learn that a formula is a special kind of condition that may be used to solve many problems of the same kind. It is important that the formulas be developed using techniques similar to those outlined in the text, so that pupils form a bridge of understanding connecting their work with formulas with their previous mathematical learnings.

If the work with trigonometry has been omitted previously, the trigonometric formulas in this lesson may be omitted.

Lesson 175 — The use of Formulas

In this lesson students apply the formulas that they developed in the last lesson to the solution of practical problems. Teachers may wish to develop solutions to problems in addition to those provided in the text before assigning the exercises. The problems involving trigonometry should be omitted if the students have not studied this topic earlier.

Lesson 176 - 177 — Business and Science Formulas, Perimeter and Area Formulas

The teacher should expect to spend some extra time on these lessons to ensure that they are covered in detail. All students leaving Grade IX should have a clear understanding of

how mathematics is applied to everyday problems. Many teachers supplement the material provided in the text. Teachers should ensure that the terminology of business and science used in the problems is clearly understood before problems are assigned.

Applying Mathematics, pages 49 - 50 should not be ignored. It will be necessary to examine the **Checking Up** exercises in detail in order that questions involving concepts which have not been studied by the students can be eliminated from the tests before they are assigned.

Unit 15 — Plane and Space Geometry

Lesson 185 — Inscribed polygons

Students were introduced to constructions in Unit 9 of the Grade VIII program. It is important to remember that inscribed polygons have vertices in the circle. The sides are not necessarily congruent. Regular polygons have congruent sides and angles and are constructed as inscribed polygons. Before starting this unit it is advisable to review basic constructions such as constructing the perpendicular bisector of a line segment and constructing the bisector of an angle.

Lesson 186 — Proofs Involving Congruent Triangles

Before teaching this lesson the teacher would be well advised to study carefully the **Content Overview** section on page 68 of the *Teaching Guide.* Although the proofs involving congruent triangles are not required for examination purposes, experienced teachers have found them valuable in the development of this lesson. If the deductive proofs are used, a more satisfactory development may be obtained through the use of overlays on the overhead projector. If commercial overlays are not available, the overlays can be made by using two or three sheets of acetate film.

Lesson 187 — The Pythagorean Property

The Pythagorean property can be developed informally by using a few specific illustrations. Be sure that students understand that the variables in $c^2 = a^2 + b^2$ represent the measures of the sides of the triangle, not the vertices. Note that the proof of the theorem using similar triangles is not included in the minimum program. If this proof is taught, the teacher may find that less capable students will have difficulty picking out the three triangles in the diagram.

Lesson 188 — The Pythagorean Property and Problem Solving

The problems in this lesson reinforce the concepts developed in Lesson 180 and illustrate some of the many practical applications of the Pythagorean property. Students visualize the problems more easily if they first sketch a triangle to represent the problem situation.

Lesson 189 — Circumference of the Circle and the Number π

This lesson will require two class periods. Have students practice constructions of inscribed polygons within congruent circles. If the perimeters are calculated using the formula $p = ns$ it will become evident that the perimeters increase as **n** increases, and that the perimeter approaches a limit, which is the circumference of the circle. Students can arrive at their own approximations of **pi** using tin cans of different sizes, string to measure the circumferences, and a ruler or measuring tape to measure the diameters. The quotient of the measures of the circumference and the diameter of any circle is an approximation of the irrational number **pi.**

Lesson 190 — Area of the Regular Polygon and Area of the Circle

Consult STM 3 *Teaching Guide,* pages 75 - 77, before teaching this lesson. Since the area of a regular polygon is derived using the formula for the area of the triangle, it is suggested that this formula be reviewed early in the lesson. Demonstrate that the area of any regular polygon with **n** sides is the area of a region formed by the union of **n** congruent triangular regions. Give the class practice in constructing apothems of polygons and show how they are used in computing area. An overhead projector is useful in this lesson. You might use it, for example, to show that as the number of sides increases the apothem of an inscribed polygon approaches its limit which is the radius of the circle. A peg-board is also an aid in the presentation of this lesson.

Lesson 191 — Lines and Planes in Space

It is suggested that this lesson be begun with a review of a few of the ideas about points, lines, and planes studied in STM 1. Since the lesson concerns points, lines, and planes in **space** it is good practice to illustrate using pieces of cardboard, string, etc. in the space of the classroom. Some students will find the illustrations in the text difficult to follow. The main point of the lesson is that the shortest distance between two parallel planes is a line segment perpendicular to the planes. The floor and ceiling, opposite walls, the desk top and the floor, all suggest parallel planes.

Lesson 192 and 193 — Right Prisms and Right Circular Cylinders, Surface Areas of Right Prisms and Right Circular Cylinders.

Several new terms are introduced in Lesson 192. The cardboard models for which patterns are provided on page 83 of the *Teaching Guide,* are excellent aids to the presentation of this lesson. If the students make their own models they grasp the idea of surface area more easily. A peg-board can be also used to construct three-dimensional models. Lay a board flat and fit dowels in a perpendicular position. Use string to form the figure.

Lesson 194 — Volumes of Right Prisms and Right Circular Cylinders.

The approach to the study of volume is similar to the approach used in the study of area, except that the basic unit is now a unit cube. Spend some time reviewing the measures used for distance, area, and volume. Students frequently have difficulty in distinguishing among the different kinds of measures. The basic formula from which all others are derived is $V = Bh$, where B represents the area of the base of the figure. It is wise to make frequent use of this formula in lesson presentation and to encourage students to use it at first when solving problems.

Lesson 195 - 196 — Regular Pyramids and Right Circular Cones, Volumes of Regular Pyramids and Regular Cones.

Lesson 195 requires that students become acquainted with a number of new terms. The students learn these terms easily if they are taught and used while the students are preparing cardboard models. Patterns are provided on pages 87 - 88 of the *Teaching Guide*. If students use the suggested dimensions they can use the models in connection with the work in Lesson 196. It can be demonstrated that it takes three cones of sand to fill the cylinder, for example.

As in Lesson 194, there is a basic formula from which more specific formulas for volume can be derived. In this case the formula is $V = \frac{1}{3} Bh$.

Lesson 197 — The Sphere

In STM 1 a standard description was developed for the circle. When defining a sphere the same standard description is used except that the universe is now space rather than a plane. The different figures produced by changing universes can be demonstrated by swinging an object attached to a piece of string, first in a single plane, and then in many different planes. Review the circle and then relate it to the sphere. When discussing great circles and small circles it is helpful to use a globe and to cut circles from pieces of cardboard so that the cardboard sheets fit on to the globe.

Lesson 198 — Surface Area and Volume of a Sphere

A crude approximation of the formula for the surface area of a sphere can be obtained by considering the area to lie between two limits. The upper limit is the surface area of the cube in which the sphere is inscribed. The lower limit is the surface area of a cube inscribed in the sphere. (A new basketball and its container provides a good illustration of the first situation). A similar approach is used to find an approximation of the formula for the volume of a sphere.

Note: A large number of formulas are introduced in this unit. Periodic reviews are essential to minimize confusion.

Unit 16 — Vectors

Only two lessons, 205 and 206, are included in the minimum program. For classes of good students Lessons 199 through 204 may be presented as enrichment. It is suggested that the lessons be treated from the standpoint of indirect measurement and scale drawings. The problems involving distance and direction, and forces or velocities have direct application to science. It should be remembered that students may not have studied trigonometric rate pairs in Grade VIII.

Lesson 205 — Addition and Subtraction of Rational Numbers

The rational numbers were first introduced in Grade VIII, Unit 10. As they studied this unit, students learned to add, subtract, multiply and divide rational numbers. Before teaching Lessons 205 and 206 it is suggested that students be assigned **Checking Up** tests 183 through 188, page 156. The teacher may use the results of these diagnostic tests to plan her approach to Lessons 205 and 206. A useful summary of the applicable work in Unit 10, Grade VIII, is provided on pages 113 - 116 of the *Teaching Guide.* Teacher-prepared worksheets may be used from time to time to maintain skills while Unit 17 is being taught.

Lesson 206 — Multiplication and Division of Rational Numbers

Before teaching this lesson read pages 117 - 119 of the *Teaching Guide.* Experience has shown that a detailed review of the Grade VIII work may be necessary. The comments concerning worksheets made in discussing Lesson 205 apply to this lesson also. The worksheets are especially valuable for slow learners. See page 119 of the *Teaching Guide* for other suggestions concerning the meeting of individual differences.

Unit 17 — The Real Number System

Lesson 211 — The Field of Rational Numbers

This lesson is a synopsis of the development of number systems and their properties. It is suggested that the class be guided through a review of the natural number system, the system of rational numbers of arithmetic, and the rational number system. It is suggested that wall charts showing the characteristics of each number system be developed with assistance from the students. Pages 129 - 131 of the *Teaching Guide* provide valuable background information and teaching suggestions.

Lesson 212 — The Field of Real Numbers

In this lesson students review a number of ideas about the real numbers that they have been exposed to earlier, and then learn that the set of real numbers, together with certain operations and properties of operations, forms a field. Again the preparation of a wall chart has been found helpful. It is strongly suggested that students be taught how to locate the

approximate position of such values as $\sqrt{10}$ on a co-ordinate graph. See pages 131 - 133 of the *Teaching Guide* for further suggestions.

Lesson 213 — A Finite Field

The new material in this lesson is not part of the minimum program, but the **Keeping Skillful** exercises page 200, provide a good review of the properties of real numbers and gives additional computational practice.

Lesson 214 — Theorems of the Real Numbers

In this lesson a distinction is made between properties that are postulated and properties that can be established as theorems once the postulates have been accepted. The *Teaching Guide* provides an overview of the content on pages 136 - 138. The proofs of theorems developed in the text are not part of the minimum program. They can be used for enrichment material.

Lesson 215 — The Ordered Field of Real Numbers

In this lesson students study five postulates of the "less than" relationships. They learn that the field of real numbers, together with these order postulates, forms an example of an ordered field. The *Teaching Guide* offers useful suggestions, pages 138 - 139.

Lesson 216 — Order Theorems of the Real Numbers

In this lesson the students learn to use order postulates to prove a few selected real number theorems. Although the proofs themselves are not required for the minimum program, average students do not find them difficult to follow. With slower pupils the conclusions can be developed through the use of examples. Read pages 140 - 141 of the *Teaching Guide* for other suggestions.

Lesson 217 — Examples of Indirect Proof

This lesson is omitted from the minimum program. If the material is included, the teacher will find the material on pages 142 - 144 of the *Teaching Guide* useful.

Lesson 218 — Simple and Compound Conditions in One Variable

This lesson will require two periods. Part of the first lesson could be used to review those portions of Units 10 and 11 that deal with solving conditions by deriving a chain of equivalent conditions leading to a condition whose solution set is easily found by inspection. In this lesson each condition in the chain is derived by applying definitions, postulates or theorems of the real numbers. It is important that students be able to justify each condition in the chain. This lesson is a good place to review the graphing of intervals and interval notation. Students studied intervals previously in Unit 12, STM 2.

Lesson 219 — Conditions in Two and Three Variables

This lesson builds on the work of Unit 13, STM 3, involving direct and inverse variation. The lesson provides an excellent opportunity to review the use of charts and co-ordinate graphs to find solution sets of compound conditions in two or three variables. An overhead projector is very useful for projecting a grid on to the blackboard which can be used for graphing.

Lesson 220 — Problems Involving Simple and Compound Conditions

It is important that this lesson be taught thoroughly, with particular attention given to careful reading of the problems and to the proper methods of setting up compound conditions and their solutions. The selection of appropriate formulas provides a good review. Note the comments concerning "Auxiliary conditions" given on page 149 of the *Teaching Guide.*

It is suggested that the problems in the **Applying Mathematics** section, page 236, be assigned two or three at a time at intervals throughout Unit 18. This procedure serves to maintain skill in problem solving techniques. Some of the questions in Test 204, page 239, can be utilized in the same way.

The **Checking Up** tests, pages 237 - 239, are useful self-tests for pupils. These too, can be assigned from time to time during the study of the next unit.

Unit 18 — Logic and the Nature of Proof

This unit introduces students to the principles of elementary logic. The minimum program includes Lesons 221, 222, 223, 224, 225 and 227 only. Since the material included in this unit is not included in most traditional texts, the teacher who is teaching this course for the first time may have to spend considerably more time than usual in lesson preparation. Teachers of experimental classes have found the background material and teaching suggestions given in the *Teaching Guide* most helpful in this regard.

Lesson 221 — The Connections "and", "or", and "not"

In this lesson students determine the truth value of compound statements involving the logical connectives "and", "or", and "not". Truth tables are introduced for the first time. Teachers have found it helpful to produce charts which are kept on display throughout the unit.

The use of Venn Diagrams assists in clarifying relationships. For example, the connective "or", meaning "one or the other or both", can be illustrated by the union of two sets.

Lesson 222 — The Connective "if . . . . . then".

Students are already very familiar with the form of "if . . . . then" statements, because they are frequently used in everyday speech. It is wise, therefore, to begin this lesson by

examining in detail conditionals provided by the students themselves. The class will probably work out the logical principles involved if guided by teacher questioning. The students can then turn their attentions to statements that are not related in any way, and finally can examine the situation in the abstract using truth tables. Good suggestions for instructional procedures are given on pages 155 - 156 of the *Teaching Guide.* The "if . . . . . then" connective is basic to the concept of direct proof.

Lesson 223 — The Converse and The Contrapositive

In this lesson students learn to form the converse and contrapositive of a conditional, and they learn how their truth values compare. Again it is suggested that the teacher start with statements that refer to experiences familiar to the students, and progress gradually to the abstract. Students should have some familiarity with the idea of converse, since they studied it in earlier grades. The concept of contrapositive takes more time to develop.

Lesson 224 — The Connective "if and only if"

The biconditional is introduced in this lesson and students learn something of its importance in mathematics. Because students will often have occasion to supply this idea to the solving of conditions, and because the basic idea is not so easily grasped by students as one might think, this lesson should be developed slowly and with thoroughness. It is essential that students understand well, for example, the difference between "if" and "only if".

The questions included in the **Applying Mathematics** section, pages 256 - 257 are good review problems. If by this time the problems at the end of Unit 17 have been completed, these questions can be assigned.

Lesson 225 — Implication

Students learn how to determine when a conditional is an implication. They study rules of logic that are most often used in proving theories. Teachers are advised to study carefully the section of the *Teaching Guide* that pertain to this lesson. Applications can be made to proofs of theorems in Unit 17.

Lesson 226 — Proof

This lesson is not included in the minimum program.

Lesson 227 — Negation

This lesson teaches students to form negations of compound conditions. The pupils learn that a statement and a negation of it have opposite truth values. The lesson prepares students for indirect proof.

Tests, Problems and Exercises

All the **Keeping Skillful** exercises found in this unit provide valuable review. They are found on pages 254, 267, and 282.

The **Applying Mathematics** section on page 287 is a valuable exercise. The business mathematics in this section should not be neglected. Note that the paragraph following Question 8 is the only place in the text that the formula for compound interest is given.

Checking Up tests, 208 to 215 should be assigned to provide individual and class review. Some of the problems in Test 215 could be carried over into Unit 19.

Unit 19 — Algebraic Expressions

Unit 19 is concerned almost entirely with algebra. It is important that the teacher regulate his time in such a way that he is able to spend at least twelve weeks on this unit. Exercises in addition to those provided in the text will be required in most lessons. The other authorized textbook, *Exploring Modern Mathematics, Book 3,* is a good source of extra questions.

The unit develops procedures used to simplify work with real numbers and variables. The students learn to simplify expressions, to perform operations upon polynomials and rational expressions, and to factor. The unit provides understanding and skills necessary for future work in algebra.

Lesson 232 — Integral Powers

The students study seven properties concerning integral powers of real numbers. The properties are accepted intuitively and in most cases have been studied previously. The *Teaching Guide* indicates where these properties were treated originally. No proofs are required.

Lesson 233 — Terminology and Agreements Concerning Expressions

This lesson introduces precise definitions, which are used throughout Unit 19. Students should have a good understanding of the terms before they are asked to verbalize the definitions. The *Teaching Guide* provides a good discussion of the definitions on pages 182 and 183.

Lesson 234 — Polynomials

The terms, "polynomial", "monomial", "binomial" and "trinomial" are introduced in this lesson along with the method of putting polynomials in standard form. The development of the lesson presented in the text is rather lengthy. The teacher should feel free to adjust his presentation. Simple, precise notes have proven beneficial to students.

Lesson 235 — Sums of Polynomials

The students learn to find the sum of two or more polynomials and to express the sum in a simplified form. These

learnings are accomplished step by step through the application of properties and theorems with which the students are familiar. The teacher should acquaint himself thoroughly with the steps and proofs before teaching the lesson. The students learn that each step can be justified, that the sum of two or more polynomials is a polynomial, and that addition is closed in the set of polynomials. Note the suggestions for providing for individual differences given on page 190 of the *Teaching Guide.*

Lesson 236 — Difference of Polynomials

The process of subtracting polynomials is broken down into a number of meaningful steps. Students are shown that each step in the process can be justified, using definitions and properties which they already know. The procedures are outlined in detail on pages 190 - 192 of the *Teaching Guide.* It is not always necessary to have students include every step in the process if they have understanding. Constant drill is necessary to develop a high degree of skill.

Lesson 237 — Products and Factoring Involving Monomials

The lesson teaches the method of finding the product of a monomial and a polynomial involving two or more terms and, the method of factoring polynomials that have a common monomial factor. The distributive property is the basis for the lesson. Students must realize that the additive inverse of each common factor is also a common factor. This is the first instance that students learn to find the highest common factor of a polynomial which involves constants and variables. Students should be provided with a number of simple drill exercises at the end of the lesson before they are asked to attempt the exercises in the text.

Lesson 238 — Products of Binomials

In this lesson students are taught to find the product of two polynomials through the use of the distributive property. The procedure for finding the product of two binomials is introduced as a special case of the above. The procedures are outlined on page 195 of the *Teaching Guide.* In the latter part of the lesson the students reverse the process to find the binomial factors of a polynomial. Polynomials of the first degree only are studied in this lesson.

Lesson 239 — Special Products and Their Factors

The students learn to factor two important types of expressions: the difference of squares and a trinomial that is a perfect square. The teacher should acquaint himself with the procedures outlined in the *Teaching Guide* before teaching this lesson. The lesson is time consuming. Extra exercises will be necessary. Be prepared to teach the factoring of expressions like $9t^2 - (r^2 - 4r + 4)$.

Lesson 240 - 241 — Trinomial Products and Factoring, More About Trinomial Products and Factoring.

These two lessons are concerned primarily with the factoring of trinomials. Lesson 240 deals with the products of binomials like $x + b$ and $x + c$. Lesson 241 deals with the products of binomials of the form $ax + b$ and $cx + d$. In each case the students find the products first and then reverse the process to find the factors. A good deal of drill in addition to the exercises in the text is necessary.

Lesson 242 — This lesson is not included in the minimum program.

Lesson 243 — Rational Expressions

It is important that students develop a clear understanding of the definition of a rational expression. In order to develop this understanding they must review operations and properties studied previously. It is suggested that the teacher study pages 210 - 212 of the *Teaching Guide* thoroughly before teaching the lesson. The material concerning meaningful replacements for variables in rational expressions has important implications in later grades.

Lesson 244 — Reduction of Quotients

This lesson involves the use of a number of operations which have been taught previously. It involves factoring, multiplying, and dividing monomials, binomials and polynomials. Several properties are reviewed and used throughout the lesson. Extra drill sheets are essential at the end of this lesson to supplement the material in the text.

Lesson 245 — Multiplication and Division of Quotients

Five theorems concerning quotients and integral powers are applied to the multiplication and division of quotients. The average class is not required to prove these theorems. Simple exercises should be prepared by the teacher and given before assigning some of the exercises in the text. Extra drill is essential.

Lesson 246 — Addition and Subtraction of Quotients

The teacher is well advised to begin this lesson with a review of the procedures used in adding and subtracting fractions, with particular attention given to expressions of the form $\frac{1}{2} + \frac{1}{3} + 2\frac{1}{4} - \frac{5}{8}$. Students do not experience serious difficulty adding and subtracting quotients with monomials as denominators, but when the expressions become more complicated, like those in Displays 7, 8 and 9, the teacher should expect that students will encounter difficulties. Student problems can be reduced by anticipating problem areas and providing work sheets which contain exercises more simple than those in the text.

The teacher should expect to use at least three periods on this lesson. Learning to divide when the dividend is a third-degree polynomial is a lesson in itself. The text contains only four exercises so extra drill is necessary.

Lesson 247 — The Square Root of a Number

This lesson discusses a number of important properties involving square root. The students learn the application of these properties, learn what is meant by the term "principal square root", and learn the technique commonly referred to as "rationalizing the divisor". The lesson will probably require at least two class periods.

Lesson 248 - 249 — Principal Roots and Rational Powers, Algebraic Expressions

These lessons are not included in the minimum program.

Unit 20 — Equivalent Conditions in One Variable

This lesson is a review of previous work with conditions for equality. The students learn to classify conditions as polynomial, rational, or algebraic. They are required to apply their knowledge of equivalent conditions and the conditional and biconditional.

The **Content Overview** section of the *Teaching Guide,* page 190, indicates how this first lesson is linked with those that follow. Note the sections which constitute the minimum program for this unit. The number of periods used to teach this and subsequent lessons and the depth to which each concept is studied should be influenced by the time available.

It is suggested that this lesson be introduced with a review of the properties used to form chains of equivalent conditions. The charts prepared earlier could be used for this purpose. The overhead projector can be utilized effectively during the presentation of this lesson.

Lesson 251 — The Linear Condition $ax + b = 0$

Conditions of the form $ax + b = 0$ occur frequently in the study of mathematics and science. In this lesson students learn a convenient method for finding solutions of conditions of this kind. Again they utilize their skill in forming chains of equivalent conditions. It is important that each step in the process be justified.

Lesson 252 — The Linear Conditions $ax+b < 0$ and $ax+b > 0$

This lesson teaches a method of solving linear conditions for inequality in one variable. Many of the ideas used in the previous lesson apply here. It is suggested that a review of the sum property of "less than" and the positive and negative-multiplier properties of "less than" be reviewed early in the lesson. Note that a portion of this lesson is not included in the minimum program. Other suggestions for instruction are given on pages 240 - 241 of the *Teaching Guide.*

Lesson 253 — Compound Linear Conditions

In this lesson students apply what they have learned in the two previous lessons to the solution of compound linear conditions in one variable. Since the lesson involves the use of a number of previous learnings, it may be necessary to spend considerable time on review. The lesson will require more than one period for most classes. Refer to the instruction notes on pages 242 - 243 of the *Teaching Guide* while preparing the lesson.

The exercises in the **Applying Mathematics** section, page 393, can be assigned two or three at a time during the teaching of the unit to provide continuous review.

Lesson 254 — Absolute Value

This lesson introduces the important concept of absolute value. The students study the definition, some properties, and the symbolization of absolute value. The idea is not entirely new to students; reference was made to it in previous grades. However, the treatment in this lesson is more formal and more complete.

Because the measure of a segment is always a positive number, the idea of absolute value can be related to it. Care must be taken to avoid any confusion that might arise between the idea of "absolute difference" of two real numbers and the idea of the "absolute value" of a single number. See page 244 of the *Teaching Guide.*

Lesson 255 — Conditions Involving Absolute Value

The **Content Overview** and **Instruction Notes** sections on page 245 of the *Teaching Guide* offer valuable suggestions for the teaching of this lesson. This lesson includes practice with skills developed earlier. It is suggested that more than one class period be set aside for this lesson.

Lesson 256 — Linear Condition in Problem Solving

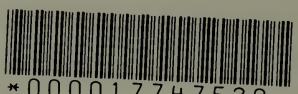
The problems in this lesson are more complex than many studied previously. The teacher should be prepared to present examples of each type and their solutions. Because the problems frequently involve more than one unknown quantity, while the conditions developed contain only one variable, it is important that pupils be required to state precisely what number or numbers the placeholder represents before forming a condition. This lesson will require more than one period.

Units 21 and 22 — Relations and Functions, Linear Functions and Systems of Conditions (Optional for an interim period)

The material in these two units has been removed from the minimum program for an interim period. However, those teachers who have time available at the end of the year would be well advised to teach some or all of the lessons. The material unifies concepts previously learned and gives students practice in finding solution sets of compound conditions in two variables.

Lessons 267 - 270 are particularly valuable. These lessons deal with systems of conditions, a topic that is given rather extensive treatment in the *Exploring Modern Mathematics* series. The work with graphing and problem solving is a fine review and provides an excellent foundation for Grade X. The EMM 3 text is a good source of additional drill exercises.

QA 14 C22 A3 A3153 1967
ALBERTA DEPT OF EDUCATION
CURRICULUM GUIDE. GRADE IX
MATHEMATICS --
39841046 CURR HIST



Date Due

QA 14 C22 A3 A3153 1967
Alberta. Dept. of Education.
Curriculum guide, grade IX
mathematics. -
39841046 CURR HIST

CURRICULUM GUIDE

For Reference

NOT TO BE TAKEN FROM THIS ROOM

EXCEPT WITH LIBRARIAN'S PERMISSION